

SFB Colloquium

TIME:

25 Oct 2011, 16:00 - 19:00

LOCATION:

Konrad-Zuse-Zentrum für Informationstechnik Berlin Takustr. 7 14195 Berlin-Dahlem

PROGRAM:

16:00 - 17:00 Prof. Dr. Igor Dolgachev

Exceptional Lie groups and algebraic geometry

Title: Exceptional Lie groups and algebraic geometry. Abstract: Does Nature use Exceptional Lie groups to express its laws of physics? This question is left to physicists to decide. However, their ubiqueness in many areas of mathematics has no doubts. In my talk I will discuss some examples of appearance of exceptional groups in classical and modern algebraic geometry. For example, the complex Lie group of type E_6 was constructed by E. Cartan as the group of linear symmetries of a certain homogeneous cubic form in 27 variables corresponding to 27 lines on a nonsingular cubic surface in three-dimensional projective space. A similar construction of the group of type E 7 involves a guartic homogeneous form in 28 variables corresponding to bitangents of a plane quartic curve. Another examples are the McKay correspondence for finite subgroups of SU(2) and the conjectural existence of the Holy Grail family of Calabi-Yau manifolds parameterized by a Hermitian symmetric space of

type E_7

17:00 - 17:30 COFFEE BREAK

17:30 - 18:30 **Prof. Dr. Bernd Ammann**

The kernel of the Dirac operator

Contact: Humboldt-Universität zu Berlin . Institut für Mathematik SFB 647 . Unter den Linden 6 . 10099 Berlin Tel. +49 30 2093 1804 . Fax. +49 30 2093 2727 sfb647@math.hu-berlin.de Title: The kernel of the Dirac operator.

We assume that \$M\$ is a compact connected oriented spin manifold equipped with a riemannian metric \$g\$. This setting allows to define spinors and a Dirac operator \$D_g\$ which is a first order elliptic differential operator acting on spinors. The spectrum of \$D_g\$ is real and discrete, and the eigenvalues can physically be interpreted as possible energy states

of a fermion.

In the talk we are mainly interested in the kernel of the Dirac operator.

The index theorem by Atiyah and Singer gives some information about the kernel, in particular it yields a lower bound on the dimension of this kernel. This lower bound is topological, i.e. it does not depend on \$g\$, but the kernel of \$D_g\$ does depend in general. Two conjectures are natural:

(1) For generic metrics the dimension of the kernel of the Dirac operator is as small as allowed by the index theorem.

(2) For special metrics the dimension is arbitrarily large.

In a collaboration with Emmanuel Humbert and Mattias Dahl we proved (1), building on previous work by Christian Bär and Mattias Dahl. We will give a new proof which yields a ``local version'' (and thus stronger) version of Conjecture (1), and which uses the unique continuation property of the Dirac operator. We will also sketch the status of Conjecture (2) which has partially been verified.